Free Convection Boundary Layer Flow of a Rotating MHD Fluid past a Vertical Porous Medium with Thermal Radiation

Amos Emeka, Amadi Chukwuemeka Paul & Nwaigwe Chinedu
Department of Mathematics,
Rivers State University, Port Harcourt,
Nigeria
amos.emeka@ust.edu.ng

Abstract
This work investigates the free convection boundary layer flow in a rotating MHD fluid past a vertical porous medium with thermal radiation. The dimensionless coupled governing boundary layer partial differential equations under the Boussinesq and Rosseland approximations are transformed into ordinary differential equation by the perturbation technique. The solutions for momentum, energy and concentration equation are solved analytically. The graphical analysis of variation in fluid velocity, temperature and concentration are displayed. The result shows that the increase in magnetic, Schmidt number, chemical reaction and rotation parameter decreases the velocity of flow in the system while the increase in thermal radiation leads to increase in velocity. Increase in chemical reaction parameter decreases heat transfer while it enhances the mass transfer.

Keywords: *Free convection, MHD, Rotation, thermal radiation, Porous medium*

1. Introduction

The concept of free convection boundary layer flow in a rotating MHD fluid past a vertical porous plate has application in geophysics, petrochemical engineering, meteorology, oceanography and aeronautics. The study of magnetic field effects on free convection Newtonian fluid flow is also used in the study of electrolytes. The effects of thermal radiation on free convention flow plays an important role in several scientific and industrial processes such as thermo-nuclear fusion, furnace design, glass production solar power technology research. Many scientists and technologists have developed interest in the study of MHD convective flows. For example, Alagoa *et al* (1999) investigated the problem of magnetohydrodynamic free convection flow, with radioactive heat transfer in porous media subject to time-dependent suction of an incompressible and optically transparent medium by making fairly realistic assumptions. Chamkha (2000) studied thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink. Israel-Cookey *et al* (2003) studied the influence of viscous dissipation and radiation on unsteady MHD free convective flow past on infinite heated vertical plate in a porous medium with time-dependent suction. Ogulu and Amos (2005) investigated the problem of suction/injection on free convective flow of a non-Newtonian fluid past a vertical porous plate. Adopting a series expansion technique about a small parameter Ec and making fairly realistic assumptions, the coupled non-linear partial differential equations are decoupled and expressions for the temperature, velocity, skin-friction and rate of heat transfer are obtained. Chin *et al* (2007) studied on the effect of variable viscosity on mixed convection boundary layer flow over a vertical surface embedded in a porous medium. Mebine and Adigio (2009) investigated the exact solution of unsteady free convection flow with thermal radiation past a vertical porous plate with Newtonian heating, employing the technique of Laplace transforms in deriving the solutions. Mahmoud (2009) also considered thermal radiation effect on unsteady MHD free convective flow of an electrically conducting fluid past an infinite vertical porous plate taking into account viscous dissipation. Afify (2009) Studied on similarity solution on MHD effects on free convective heat and mass transfer over a stretching surface considering suction or injection. Mohamed (2009) studied double-diffusive convection-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effect. Israel-Cookey *et al* (2012) investigated the problem of magnetohydrodynamic free convection and oscillatory flow of an optically thin fluid bounded by two horizontal porous parallel walls under the influence of an externally imposed magnetic field. Baoku *et al* (2012) studied the influence of thermal radiation on a transient MHD couette flow through a porous medium. Salem and Fathy (2012) studied the effects of variable properties on MHD heat and mass transfer flow near a stagnation point towards a stretching sheet in porous medium with thermal radiation. Reddy (2014) discussed the analytical study for the problem of mixed convection with thermal radiation and first-order chemical reaction on magnetohydrodynamic boundary layer flow of viscous, electrically conducting fluid past a vertical permeable surface embedded in a porous medium. Nayak (2015) worked on heat and mass transfer effects on boundary layer flow through porous medium of an electrically conducting viscoelastic fluid subject to transverse magnetic field in the presence of heat source/sink and chemical reactions. Seth *et al* (2015) investigated the unsteady hydromagnetic free convection flow of a viscous, incompressible, electrically conducting, optically thick radiating and heat absorbing fluid past an accelerated moving vertical plate with variable ramped temperature. Seth *et al* (2011) also investigated the effect of radiation and rotation of unsteady hydromagnetic free convection flow of a viscous incompressible electrically conducting fluid past an impulsively moving infinite vertical plate with ramped temperature in a porous medium in which the exact solution of momentum and energy equations under Boussinesq approximation were obtained. Krishna and Reddy (2018) investigated the unsteady MHD free convection in a boundary layer flow of an electrically conducting fluid through porous medium subject to uniform transverse magnetic field over a moving infinite vertical plate in the presence of heat source and chemical reaction. The governing equations which were based on Brinkman's model were transformed into a two point boundary value problem which was solved numerically by Runge-kutta fourth order method with the help of shooting techniques. Raju *et al* (2018) investigated the numerical solutions of unsteady hydromagnetic free convection couette flow of a viscous, incompressible and electrically conducting fluid between two vertical parallel plates in the presence of thermal radiation, thermal diffusion and diffusion thermo. The dimensionless coupled partial differential equations for impulsive movement and uniformly accelerated movement of the plate were solved by an efficient finite method. The aim of this study investigates heat and mass transfer in a boundary layer flow and the effects of thermal radiation on the flow of a rotating MHD fluid.

2. Mathematical Formulation

Consider an unsteady free convection flow of an electrically conducting, viscous, incompressible fluid past a vertical infinite plate embedded in a porous medium in the presence of thermal radiation and chemical reaction. We consider the Cartesian coordinate system in which the x' axis vertical (along the plate) and the z' axis is normal to it. The infinite plate rotates with constant angular velocity Ω about the axis perpendicular to it. A uniform magnetic field of strength B_0 is applied perpendicular to the flow direction. The temperature and concentration of the plate varies with time about a constant mean while the free-stream temperature and concentration is constant. The interaction of the cariolis forc with the free convection sets up a secondary flow in addition to primary flow and hence the flow becomes three dimensioal. Since the plate is infinite in length, the flow variables are fucntions of z' and t' only.

With the above assumption and the Boussinesq aproximation; the governing equation for continuity, momentum, energy and concentration are as follows:-

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + w'\frac{\partial u'}{\partial z'} - 2\Omega v' = g\beta_T(T' - T_\infty) + g\beta_c(C' - C_\infty) + v\frac{\partial^2 u}{\partial z^2} - \sigma\frac{B_0^{\,2}v'}{\rho} - \frac{vu'}{Kp'} \tag{2}$$

$$\frac{\partial v'}{\partial t'} + w'\frac{\partial v'}{\partial z'} + 2\Omega u' = v\frac{\partial^2 v'}{\partial z^2} - \sigma\frac{B_0^{\,2}v'}{\rho} - \frac{vV'}{Kp'} \tag{3}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z'} - \frac{vw'}{Kp'} \tag{4}$$

$$\frac{\partial T'}{\partial t'} + w'\frac{\partial T'}{\partial z'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial z'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial z'} \tag{5}$$

$$\frac{\partial C'}{\partial t'} + w'\frac{\partial C'}{\partial z'} = D\frac{\partial^2 C'}{\partial z'^2} - K'_C(C' - C_\infty) \tag{6}$$

The appropriate boundary condition for the equations (2), (5), and (6) is given as follows;

$$u' = 0,\ v' = 0,\ T' = T'_W + \varepsilon(T'_W - T'_\infty)e^{i\omega' t'}$$
$$C' = C'_W + \varepsilon(C'_W - C'_\infty)e^{i\omega' t'}, \quad \text{at} \quad z' = 0$$
$$u', v' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty \qquad \text{as} \quad z' \to \infty \tag{7}$$

C_p – is the specific heat or the fluid, $\rho -$ is the density of the fluid, $\sigma -$ is the electrical conductivity of the fluid, K – is the thermal-conductivity of the fluid, β_T – is the coefficient of the volume expansion due to temperature or thermal expansion, β_c – is the coefficient of expansion due to concentration, Kc– is the chemical reaction, g–is acceleration due to gravity, D – is the chemical diffusion, K_p – is the permeability of the porous medium, B_0 – is the magnetic field strength, C_w, T_w– is concentration and temperature at the wall respectively. $C_\infty, T_\infty - is$ a free stream concentration and temperature respectively.

where ε is a small positive number, w_0 is the constant suction velocity, A is a suction parameter. Equation (4) determines the pressure distribution along the axis of rotation. We then invoke the Rosseland approximation for the radiative flux given as;

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial z'} \tag{8}$$

The σ^* *is* Stefan –Boltzmann constant and K^* is the mean absorption coefficient.

$$w' = -w_0(1 + \varepsilon A e^{i\omega' t'}) \tag{9}$$

where $\mathcal{E} << 1$ and ω is the frequency of oscillation.

The secondary flow is assumed small and its influence is considered only on the rotation term, hence writing $U = u + iv$

we have

$$\frac{\partial U'}{\partial t'} - w_0\left(1 + \mathcal{E}A\, e^{i\omega' t'}\right)\frac{\partial U'}{\partial z'} + 2i\Omega w_0 U = v\frac{\partial^2 U'}{\partial z'^2} - \frac{\sigma B_0^2 w_0 U}{\rho} - \frac{vUw_0}{K'_p} + g\beta_T(T - T'_\infty) + g\beta_c(C'_w - C'_\infty). \tag{10}$$

We further assume that the temperature difference within the flow are sufficiently small and therefore T'^4 can be expressed as a linear function of temperature about the free stream T_∞ using Taylor's series which on neglecting higher order terms becomes;

$$(T'^4) = 4T'(T'_\infty)^3 - 3(T'_\infty)^4 \tag{11}$$

Consequently the following equations (8) and (9), equation (5) becomes.

$$\frac{\partial T'}{\partial t} + w'\frac{\partial T'}{\partial z'} = \frac{K}{\rho Cp}\frac{\partial^2 T'}{\partial z'^2} + \frac{16\sigma^* K T_\infty'^2}{3\rho Cp k'} \quad (12)$$

The non dimensional variables for the problem are;

$u = \frac{u'}{w_0}$, $t = \frac{t' W^2_0}{v}$, $\Pr = \frac{\rho c_p v}{K}$, $K_c = \frac{v k'_c}{W^2_0}$, $R_0 = \frac{\Omega v}{W^2_0}$, $\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}$, $Gr = g\beta_T v\left(\frac{T'_w - T'_\infty}{W^3_0}\right)$,

$S = \frac{S' v}{\sigma c_p W^2_0}$, $\phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}$, $Sc = \frac{v}{D}$, $z = w_0 \frac{z'}{v}$, $K_p = \frac{W^2_0 k' p}{v^2}$, $Gc = g\beta_c v\left(\frac{C'_w - C'_\infty}{W^3_0}\right)$,
$w = \frac{w' v}{W^2_0}$,

$$M = \sqrt{\frac{\sigma B^2_0 v}{\rho W^2_0}}, \quad Rd = \frac{16\sigma^* T_\infty'^3}{3k^* k} \quad (13)$$

Therefore, using equations (9) and (13) on equation (2), (6) and (12) we have

$$\frac{\partial U}{\partial t} - \left(1 + \mathcal{E} A e^{i\omega t}\right)\frac{\partial U}{\partial z} + 2iR_0 U = \frac{\partial^2 U}{\partial z^2} - \left(M^2 + \frac{1}{k_p}\right)U + Gr\theta + Gc\phi \quad (14)$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \mathcal{E} A e^{i\omega t}\right)\frac{\partial \theta}{\partial z} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2}(1 + Rd) \quad (15)$$

$$\frac{\partial \phi}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial \phi}{\partial z} = \frac{1}{sc}\frac{\partial^2 \phi}{\partial z^2} - K_c\phi \quad (16)$$

The boundary conditions simplifies as

$$u = 0,\ \theta = 1 + \mathcal{E}\ e^{i\omega t},\ C = 1 + \mathcal{E}\ e^{i\omega t} \quad \text{at } z = 0 \quad (17)$$

$$u \to 0,\ T - T_\infty, C - C_\infty \quad \text{as } z \to \infty \quad (18)$$

The parameter entering the problem are *Rd*, R_0, *M*, *Kp*, *Kc*, *Sc*, *Pr*, *Gr*, *Gc*, which represents, thermal radiation, rotation parameter, magnetic parameter, permeability of the porous medium, Chemical reaction, Schmidt number, Prandtl number, thermal Grashof number, mass Grashof number, respectively.

The mathematical formulation obtained equation (14) – (16) together with the boundary conditions on equation (17) and (18). Hence, the mathematical formulations are now complete. We are to solve equations (14) - (16) subject to the boundary conditions.

3. Method of Solution

Equations (14) – (16) are coupled non-linear differential equations and to obtain approximate analytical solutions we adopt a perturbation of the form:

$$u(z,t) = u_0(z) + \varepsilon\left[e^{i\omega t}u_1(z) + e^{-i\omega t}u_2(z)\right]$$
$$\theta(z,t) = \theta_0(z) + \varepsilon\left[e^{i\omega t}\theta_1(z) + e^{-i\omega t}\theta_2(z)\right] \quad 19)$$
$$\phi(z,t) = \phi_0(z) + \varepsilon\left[e^{i\omega t}\phi_1(z) + e^{-i\omega t}\phi_2(z)\right]$$

where U, ϕ, and θ stand for velocity, Concentration parameter and Temperature parameter respectively. The equations in (19) are valid for small amplitude. Substituting equation (19) into equations (14) - (16) appropriately for velocity, temperature, concentration and the associated boundary conditions, gives the perturbed equations. For u_0, θ_0 *and* ϕ_0 we have:

$$\frac{d^2u_0}{dz^2} + \frac{du_0}{dz} - \left[\left(M^2 + \frac{1}{K_p}\right) + 2iR_0\right] u_0 = -Gr\theta_0 - Gc\phi_0 \quad (20)$$

$$\frac{1+Rd}{Pr}\frac{d^2\theta_0}{dz^2} + \frac{d\theta_0}{dz} = 0 \quad (21)$$

$$\frac{1}{Sc}\frac{d^2\phi_0}{dz} + \frac{d\phi_0}{dz} - K_c\phi_0 = 0 \quad (22)$$

$$u_0 = 0,\ \theta_0 = 1, \phi_0 = 1 \text{ at } z = 0$$
$$u_0 \to 0,\ \theta_0 \to 0, \phi_0 \to 0 \text{ at } z \to \infty \quad (23)$$

For u_1, θ_1 *and* ϕ_1, we have:

$$\frac{d^2u_1}{dz^2} + \frac{du_1}{dz} - \left[\left(M^2 + \frac{1}{K_p}\right) + 2iR_0 + i\omega\right] u_1 = -A\frac{du_0}{dz} - Gr\theta_1 - Gc\phi_1 \quad (24)$$

$$\frac{1+Rd}{Pr}\frac{d^2\theta_1}{dz^2} + \frac{d\theta_1}{dz} - i\omega\theta_1 = -A\frac{d\theta_0}{dz} \quad (25)$$

$$\frac{1}{Sc}\frac{d^2\phi_1}{dz^2} + \frac{d\phi_1}{dz} - (i\omega + K_c)\phi_1 = -A\frac{d\phi_0}{dz} \quad (26)$$

$$u_1 = 0,\ \theta_1 = 1, \phi_1 = 1 \text{ at } z = 0$$
$$u_1 \to 0,\ \theta_1 \to 0,\ \phi_1 \to 0 \text{ as } z \to \infty \quad (27)$$

For u_2, θ_2 *and* ϕ_2, the governing equations are the following:

$$\frac{d^2u_2}{dz^2} + \frac{du_2}{dz} - \left[\left(M^2 + \frac{1}{K_p}\right) + 2iR_0 - i\omega\right] u_2 = -Gr\theta_2 - Gc\phi_2 \quad (28)$$

$$\frac{1+Rd}{Pr}\frac{d^2\theta_2}{dz^2} + \frac{d\theta_2}{dz} + i\omega\theta_2 = 0 \quad (29)$$

$$\frac{1}{Sc}\frac{d^2\phi_2}{dz^2} + \frac{d\phi_2}{dz} + (i\omega - K_c)\phi_2 = 0 \quad (30)$$

With the boundary conditions:

$$u_2 = 0,\ \theta_2 = 0,\ \phi_2 = 0 \text{ at } z = 0$$
$$u_2 \to 0,\ \theta_2 \to 0,\ \phi_2 \to 0 \text{ as } z \to \infty \quad (31)$$

The solution of equation (20) – (31) after some long algebraic process is presented as:

$$u(z,t) = (c_1 + c_2)e^{-\lambda_6 z} - c_1e^{-\lambda_4 z} + e^{i\omega t}[(1 - \alpha_0 - B_1 - B_2 - B_3 - B_4)e^{-\lambda_{12} z} + \alpha_0 e^{-\lambda_6 z} + B_1e^{-\lambda_4 z} + B_2e^{-\lambda_{10} z} + B_3e^{-\lambda_8 z} + B_4e^{-\lambda_2 z}. \quad (32)$$

$$\theta(z,t) = e^{-\lambda_4 z} + e^{i\omega t}[(1 - C_4)e^{-\lambda_{10} z} + C_4e^{-\lambda_4 z}]\,. \quad (33)$$

$$\phi(z,t) = e^{-\lambda_2 z} + e^{i\omega t}[(1 - C_3)e^{-\lambda_8 z} + C_3e^{-\lambda_2}]\,. \quad (34)$$

3.2 Skin friction, heat and mass transfer fluxes

It is necessary to know the skin friction on the wall and the heat and mass transfer rates within the fluid. We therefore have the following:

Skin friction

$$-\left.\frac{du}{dz}\right|_{z=0} = \lambda_6(c_1 + c_2) + \lambda_4c_1 + \lambda_2c_2 + \mathrm{e}^{\mathrm{i}\omega\mathrm{t}}[\lambda_{12}(1 - \alpha_0 - B_1 - B_2 - B_3 - B_4) - \lambda_6\alpha_0 - \lambda_4B_1 - \lambda_{10}B_2 - \lambda_8B_3 - \lambda_2B_4\,]. \quad (35)$$

Heat transfer

$$\left.\frac{d\theta}{dz}\right|_{z=0} = -\lambda_4 + \mathrm{e}^{\mathrm{i}\omega\mathrm{t}}[-\lambda_{10}(1 - c_4) - \lambda_4\mathrm{c}_4] \quad (36)$$

Mass transfer

$$\left.\frac{d\phi}{dz}\right|_{z=0} = -\lambda_2 + \mathrm{e}^{\mathrm{i}\omega\mathrm{t}}[-\lambda_8(1 - c_3) - \lambda_2\mathrm{c}_3] \quad (37)$$

4. Discussion of Results

We have formulated and computed results for the problem under study. The values of the parameters adopted from the graphical presentations are shown, consequently we present our discussion.

The influence of variation in rotation parameter on the velocity is shown in Figure 1. The profile indicates that the introduction of increased rotation in the system decreases the velocity. The rotation is seen to interface with the smooth flow of the fluid, hence a decrease in the flow. Figure 2 shows the effect of variation in thermal radiation on velocity. It is observed that the increase in thermal radiation increases the velocity. The response of velocity to variation in magnetic field is depicted in Figure 3. It can be deduced from the profile that increase in the magnetic field reduces the velocity of fluid. The presence of magnetic field in an electrically conducting fluid introduces the Lorentz force which resists fluid flow. The effect of permeability on velocity is shown in Figure 4. The velocity increase at the boundary layer is as a result of increase in the permeability and peaks up at $z = 1$, and thereafter decelerates to assume the free stream velocity. The effect of chemical reaction on velocity is illustrated in Figure 5. It is noted that the increase in chemical reaction leads to decrease in velocity. The profile shows a peak close to the plate where it assumes maximum value and then decelerates to assume the free stream velocity. Figure 6 shows the effect of variation of the Schmidt number on the velocity. Increase in the Schmidt number decreases the momentum boundary layer, the consequences of this is a decrease in the velocity.

The response of the solutal buoyancy force Gc and the thermal buoyancy force Gr are illustrated in Figure 7 and Figure 8 respectively. Increase in Gc or Gr leads to increase in momentum boundary layer thickness. The effect of this is that increase in Gc or Gr enhance the fluid velocity.The influence of rotation on the temperature is shown in Figure 9. It is clearly seen that increase in rotation parameter leads to increase in temperature. Physically the rotation aids temperature distribution in the system. Figure 10 shows that the concentration of species decreases with time in the solutal boundary layer. The effect is a decrease in solutal boundary layer thickness. The influence of variation of chemical reaction on the concentration is illustrated in Figure 11. It reveals that increase in the chemical reaction decreases the concentration of species. Figure 12 shows that increase in the Schmidt number decreases the concentration of species. Increase in Schmidt number mean decrease in molecular diffusivity and leads to decrease in the concentration boundary layer. The response of concentration of species on variation in suction parameter is shown in Figure 13. The profile indicates that increased suction decreases the concentration.

Figure 14 shows the effect of rotation on the skin friction. It indicates that rotation increases the skin friction. Increase in Grashof number does not change the trend. The influence of porosity parameter on the skin friction is shown in Figure 15. Increase in porosity increases the skin friction coefficient. Figure 16 shows effect of variation of chemical reaction on heat transfer rate. Increase in chemical reaction parameter decreases the heat transfer rate as much of the energy is used up in the reaction process; hence a decrease in the heat transfers. Figure 17 shows that mass transfer is increased as the chemical reaction parameter increases with the increase in oscillation parameter exerting minimal influence. Figure 18 shows profile for mass transfer rate illustrating the effect of Schmidt number on heat transfer. It is observed that the heat transfer increases due to increase in Schmidt number with the chemical reaction showing no sufficient influence on the mass transfer rate.

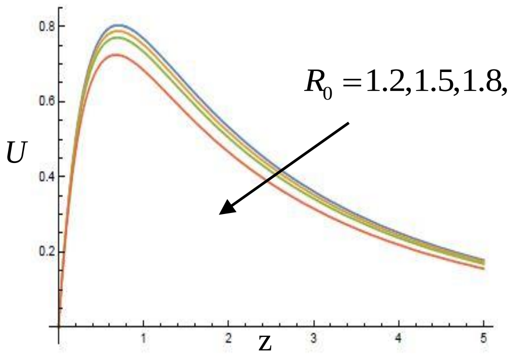

Figure 1: Velocity profiles showing effect of rotation for M = 1.5, K_p = 0.2, Gr = 5, Gc = 5, Rd = 1.5, A=1, $\varepsilon = 0.1$, Sc = 0.20, Kc = 1.5, Pr = 0.71, K = 1.5

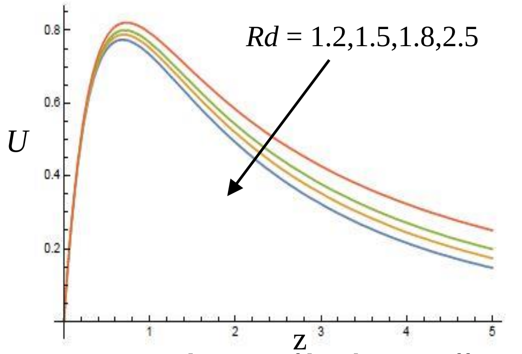

Figure 2: Velocity profiles showing effect of thermal radiation for M = 1.5, K_P = 0.2, Gr = 5, Gc = 5, R_o = 1.5, A = 1, $\varepsilon = 0.1$, Sc = 0.20, Kc = 1.5, Pr = 0.71, K = 1.5.

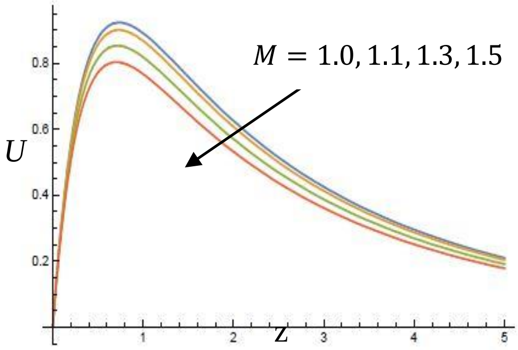

Figure 3: velocity profile showing effect of magnetic field for Rd = 1.5, K_P = 0.2, Gr = 5, Gc = 5, R_o=1.5, A=1, $\varepsilon = 0.1$, Sc = 0.20, Kc = 1.5, Pr = 0.71, K = 1.5.

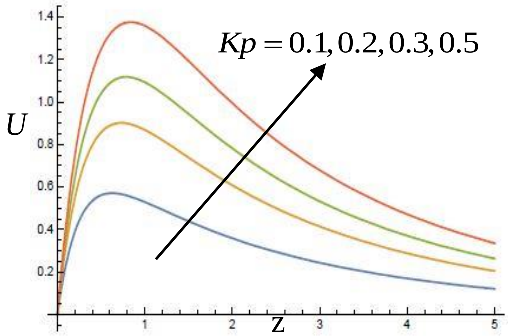

Figure 4: Velocity profiles showing effect of permeability M = 1.5, K_p = 0.2, Rd = 1.5, Gr = 5, Gc = 5, R_o = 1.5, A = 1, $\varepsilon = 0.1$, Sc = 0.20, Kc = 1.5, Pr = 0.71.

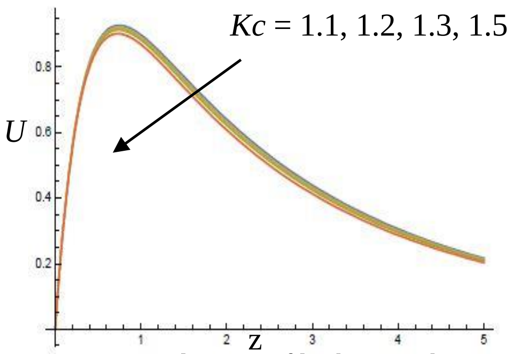

Figure 5: Velocity profile showing the effect of chemical reaction on M=1.5, Gr = 5, Gc = 5, R_o = 1.5, A = 1, Pr = 0.71, K = 1.5, K_p = 0.2, $\varepsilon = 0.1$.

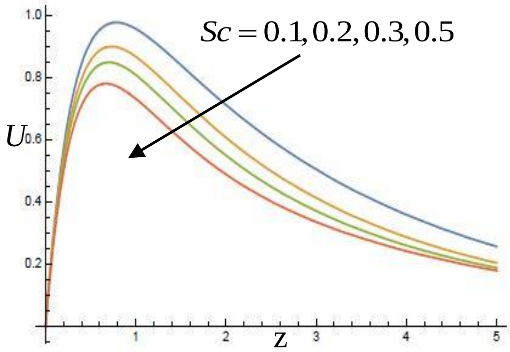

Figure 6: Velocity profile showing the effect of Schmidt number on M = 1.5, Kp = 0.2, Gr = 5, Gc = 5, Ro = 1.5, A = 1, Pr = 0.71, K = 1.5, Kp = 0.2, $\varepsilon = 0.1$

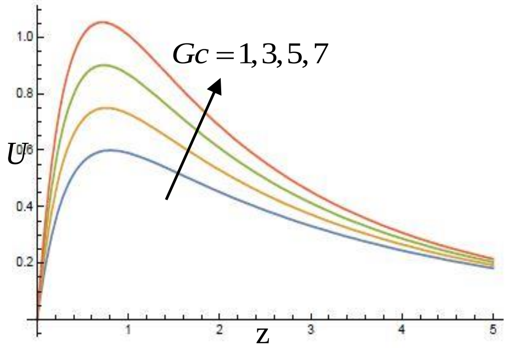

Figure 7: Velocity profile showing the effect of mass grashof number on M = 1.5, K_p = 0.2, Gr = 5, R_o = 1.5, A = 1, Pr = 0.71, K = 1.5, K_p = 0.2, ε = 0.1, Sc = 1.5

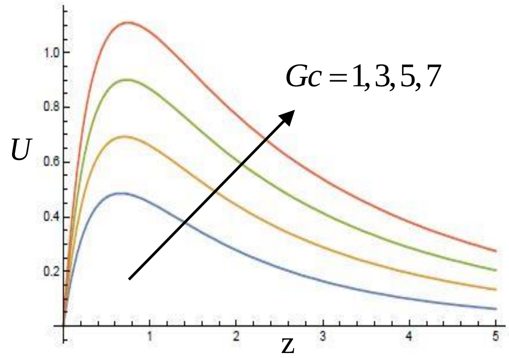

Figure 8: Velocity profile showing the effect of Thermal Grashof mumber for M = 1.5, K_p = 0.2, Gr = 5, R_o = 1.5, A=1, Pr = 0.71, K = 1.5, K_p = 0.2, ε = 0.1, Sc = 1.5

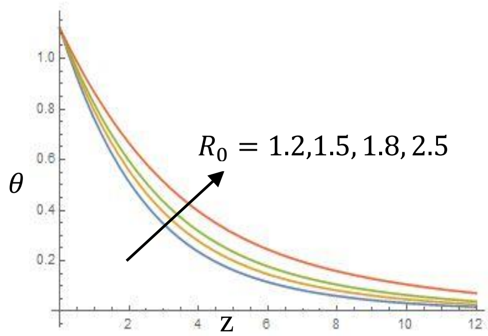

Figure 9: Temperature profiles showing effect of rotation for M = 1.5, K_p = 0.2, Gr = 5, Gc = 5, Rd = 1.5, A = 1, ε = 0.1, Sc = 0.20, Kc = 1.5, Pr = 0.71, K = 1.5

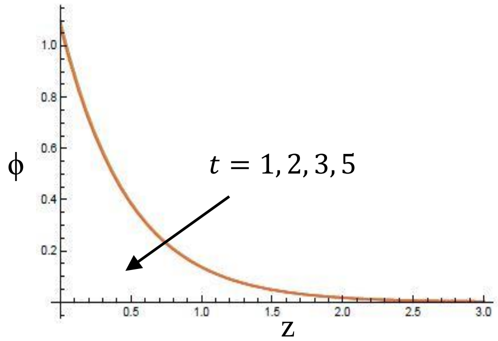

Figure 10: concentration profiles showing effect of time for Sc = 0.20, Kc = 1.5, ω = 1, A = 1, ε = 0.1

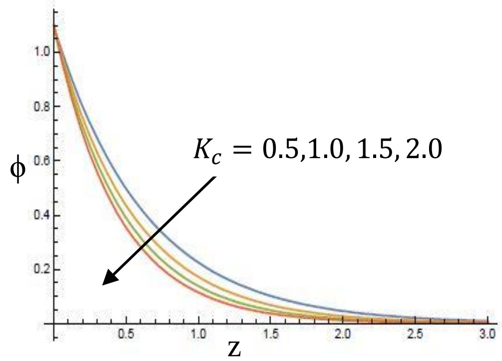

Figure 11: concentration profiles showing effect of chemical reaction for Sc = 0.20, ω = 1, A=1, t =1, ε = 0.1.

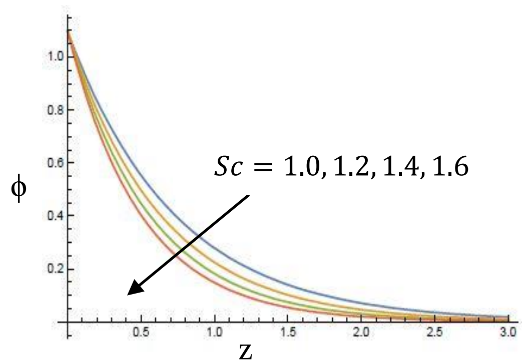

Figure 12: concentration profiles showing effect of Schmidt number for Kc = 1.5, ω = 1, A=1, t =1, ε = 0.1.

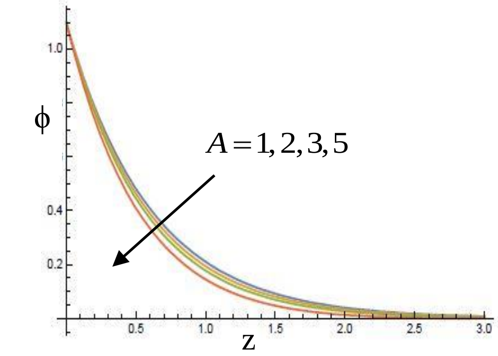

Figure 13: concentration profiles showing effect of suction parameter for Sc = 0.20, Kc = 1.5, $\omega = 1$, t = 1, $\varepsilon = 0.1$

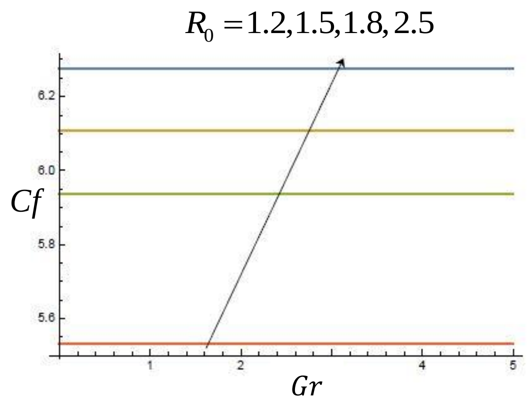

Figure 14: Skin friction profile showing effect of rotation for M = 1.5, $K_p = 0.2$, Gc = 5, Rd = 1.5, A = 1, $\varepsilon = 0.1$, Sc = 0.20, Kc = 1.5, Pr = 0.71, K = 1.5

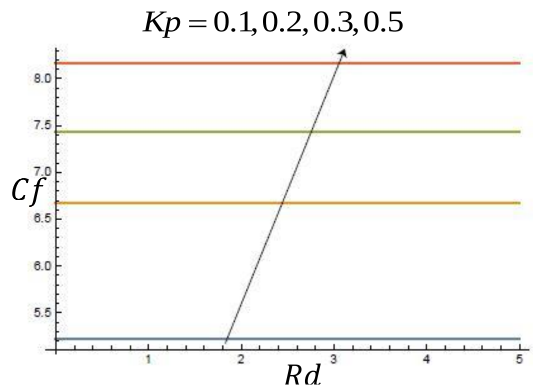

Figure 15: The skin friction profile showing the effect of permeability Kp for M = 1.5, Gc = 5, A = 1, $\varepsilon = 0.1$, Sc = 0.20, Kc = 1.5, Pr = 0.71, K = 1.5

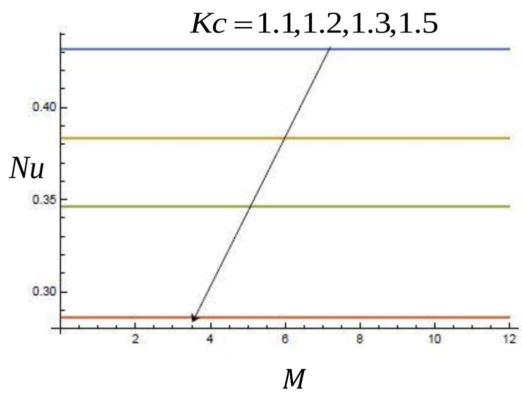

Figure 16: Heat transfer profile showing effect of chemical reaction for Pr = 0.71, Rd = 1.5.

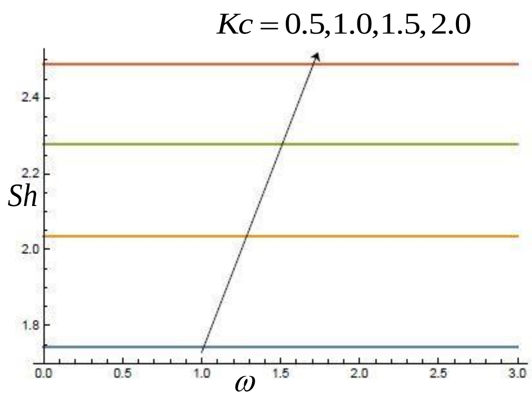

Figure 17: Mass transfer profile showing effect of chemical reaction for A = 1, $\varepsilon = 0.1$, Sc = 0.20, t = 1

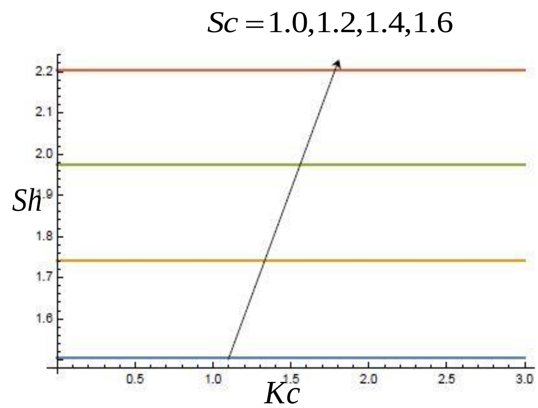

Figure 18: Mass transfer profile showing effect of chemical reaction for A = 1, $\varepsilon = 0.1$, Sc = 0.20, t = 1.

Conclusion

In this research, analysis of free convection boundary layer flow in a rotating magnetohydrodynamic (MHD) fluid past through a vertical porous medium with thermal radiation was investigated. The conclusions based on this study are as follows:

i. The increase in magnetic field, Schmidt number, chemical reaction and the rotation parameter decreases the velocity of flow in the system while the increase in thermal radiation, mass Grashof number and thermal Grashof number enhances the velocity of fluid flow.
ii. The increase in rotation parameter increases the temperature distribution of the system.
iii. The increase in chemical reaction, Schmidt number and suction parameter decreases the concentration boundary layer.

Reference

Afify, A. A. (2009). Similarity solution in MHD: effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. *Communications in Nonlinear Science and Numerical Simulation*, 14(5), 2202-2214.

Alagoa, K. D., Tay, G. and Abbey, T. M. (1999). Radiative and free convective effects of a MHD flow through a porous medium between infinite parallel plates with time-dependent suction. *Astrophysics and Space Science*, 260(4), 455-468.

Baoku, I. G., Israel-Cookey, C. and Olajuwon, B. I. (2012). Influence of Thermal Radiation on a Transient MHD Couette Flow through a Porous Medium. *Journal of Applied Fluid Mechanics*, 5(1).

Chamkha, A. J. (2000). Thermal radiation and buoyancy effects on hydromagnetic flow over an `accelerating permeable surface with heat source or sink. *International Journal of Engineering Science*, 38(15), 1699-1712.

Chin, K. E., Nazar, R., Arifin, N. M. and Pop, I. (2007). Effect of variable viscosity on mixed convection boundary layer flow over a vertical surface embedded in a porous medium. *International Communications in Heat and Mass Transfer*, 34(4), 464-473.

Israel-Cookey, C., Amos, E. and Nwaigwe, C. (2010). MHD oscillatory Couette flow of a radiating viscous fluid in a porous medium with periodic wall temperature. *Am. J. Sci. Ind. Res*, 1(2), 326-331.

Israel-Cookey, C., Ogulu, A. and Omubo-Pepple, V. B. (2003). Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction. *International Journal of Heat and Mass Transfer*, 46(13), 2305-2311.

Krishna, M. V. and Reddy, M. G. (2018). MHD Free Convection Boundary Layer Flow through porous medium past a moving vertical plate with heat source and chemical Reaction. *Material today: proceedings*, 5(1), 91-98.

Mahmoud, M. A. (2009). Thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature-dependent viscosity. *The Canadian Journal of Chemical Engineering*, 87(1), 47-52.

Mebine, P. and Adigio, E. M. (2009). Unsteady free convection flow with thermal radiation past a vertical porous plate with Newtonian heating. *Turkish Journal of Physics*, 33(2), 109-119.

Mohamed, R. A. (2009). Double-diffusive convection-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects. *Applied mathematical sciences*, 3(13), 629-651.

Nayak, M. K. (2015). Steady Free Convection Heat and Mass Transfer MHD Flow of a Radiative Micropolar Fluid in a Vertical Porous Channel with Heat Source and Chemical Reaction, Modelling. *Measurement and Control B*, 84(2), 52-79.

Ogulu, A. and Amos, E. (2005). Asymptotic approximations for the flow field in a free convective flow of a non-Newtonian fluid past a vertical porous plate. *International communications in heat and mass transfer*, 32(7), 974-982.

Raju, R. S., Reddy, G. J., Rao, J. A. and Rashidi, M. M. (2016). Thermal diffusion and diffusion thermo effects on an unsteady heat and mass transfer magnetohydrodynamic natural convection coquette flow using FEM. *Journal of computational Design and Engineering*, 3(4), 349-362.

Reddy, M. G. (2014). Thermal radiation and chemical reaction effects on MHD mixed convective boundary layer slip flow in a porous medium with heat source and Ohmic heating. *The European Physical Journal Plus*, 129(3), 41

Salem, A. M. and Fathy, R. (2012). Effects of variable properties on MHD heat and mass transfer flow near a stagnation point towards a stretching sheet in a porous medium with thermal radiation. *Chinese Physics B*, 21(5), 054701.

Seth, G. S., Kumbhakar, B. and Sharma, R. (2015). Unsteady Hydromagnetic Natural Convection Flow of a Heat Absorbing Fluid within a Rotating Vertical Channel in Porous Medium with Hall Effects. *Journal of Applied Fluid Mechanics*, 8(4).

Seth, G. S., Nandkeolyar, R. and Ansari, M. S. (2011). Effect of rotation on unsteady hydromagnetic natural convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium with thermal diffusion and heat absorption. *International Journal of Applied Mathematics Mechanics*, 7(21), 52-69.

Appendix

$\lambda_2 = \frac{-\left(sc+\sqrt{sc^2+4scK_c}\right)}{2}$, $\lambda_4 = -\frac{Pr}{1+Rd}$, $\lambda_6 = \frac{-1-\sqrt{1+4D_0}}{2}$, $\lambda_8 = -\left(\frac{sc+\sqrt{sc^2+4scD_1}}{2}\right)$,
$\lambda_{10} = \frac{-\left(D_2+\sqrt{D_2^2+4D_2 i\omega}\right)}{2}$, $\lambda_{12} = \frac{-\left(1+\sqrt{1+4D_3}\right)}{2}$, $D_0 = \left(M^2+\frac{1}{K_p}\right)+2iR_0$, $D_1 = i\omega + K_c$,
$D_2 = \frac{Pr}{1+Rd}$, $D_3 = \left(M^2+\frac{1}{K_p}\right)+2iR_0+i\omega$, $D_4 = i\omega - K_c$, $D_5 = \left(M^2+\frac{1}{K_p}\right)+2iR_0 - i\omega$,
$B_0 = \frac{A\lambda_6(C_1+C_2)}{\lambda_6^2-\lambda_6-D_3}$, $B_1 = \frac{A\lambda_4 C_1+GrC_4}{\lambda_4^2-\lambda_4-D_3}$, $B_2 = \frac{Gr(1-C_4)}{\lambda_{10}^2-\lambda_{10}-D_3}$, $B_3 = \frac{Gc(1-C_4)}{\lambda_8^2-\lambda_8-D_3}$, $B_4 = \frac{A\lambda_4 C_1+GcC_3}{\lambda_8^2-\lambda_8-D_3}$,
$C_1 = \frac{Gr}{\lambda_4^2-\lambda_4-D_0}$, $C_2 = \frac{Gc}{\lambda_2^2-\lambda_2-D_0}$, $C_3 = \frac{A\lambda_2}{\lambda_2^2-sc\lambda_2-scD_1}$, $C_4 = \frac{A\lambda_4}{\lambda_4^2-\lambda_4 D_2-D_2 i\omega}$, $\phi_0(z,) = e^{-\lambda_2 z}$,
$\theta_0(z) = e^{-\lambda_4 z}$, $U_0(z) = (c_1+c_2)e^{-\lambda_6 z} - c_1 e^{-\lambda_4 z} - c_2 e^{-\lambda_2 z}$, $\phi_1(z) = (1-C_3)e^{-\lambda_8 z} + C_3 e^{-\lambda_2 z}$,
$\theta_1(z) = (1-C_4)e^{-\lambda_{10} z} + C_4 e^{-\lambda_4 z}$, $u_1(z) = (1-\alpha_0-B_1-B_2-B_3-B_4)e^{-\lambda_{12} z} + B_0 e^{-\lambda_6 z} + B_1 e^{-\lambda_4 z} + B_2 e^{-\lambda_{10} z} + B_3 e^{-\lambda_8 z} + B_4 e^{-\lambda_2 z}$